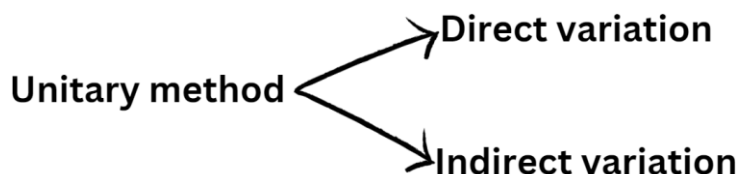# Unitary Method and Time and Work

Unitary method
- → Direct variation
- → Indirect variation

Direct variation examples:
* *More* amount, *more* things to buy
* *Less* things bought; *less* amount paid
* *More* petrol, *more* distance travelled
* *More* distance travelled, *more* time taken, *more* petrol, *more* amount paid
* *Less* time travelled, *less* distance covered, *less* petrol used, *less* amount paid
* *More* work done; *more* amount earned

Indirect variation examples:
* *More* people worked; *less* days taken to do it
* *Less* people, *more* days to consume food
* *More* speed, *less* time taken to travel
* *Less* rate of filling, *more* time to fill the tank

Direct variation (divide)	Indirect variation (multiply)
200 km travelled in 5 hours. 400 km travelled in ??? hours	200 people work for 5 hours. 400 people complete in ??? hours
More distance needs *more* time	*More* people take *less* time
Unitary 200 km —travelled in→ 5 hours 1 km —travelled in→ ???	Unitary 200 people —do work in→ 5 hours 1 person —does work in→ ???
Divide: $(5 \div 200) \times 1$ 1 km travelled in 5/200 hours	**Multiply**: $(5 \times 200) \div 1$ 1 person takes 1000 hours to work
1 km —travelled in→ $\frac{5}{200}$ hours 400 km —travelled in→ $\frac{5}{200} \div 1 \times 400 = 10$ hrs	1 person —does work in→ 1000 hours 400 people —do work in→ $\frac{1000 \times 1}{400} = 2.5$ hrs

In general:			In general:		
200 km	5 hrs		200 p	5 hrs	
400 km	x hrs		400 p	x hrs	

$$\frac{5}{200} = \frac{x}{400}$$

$$x = 10 \; hrs$$

$$5 \times 200 = x \times 400$$

$$x = 2.5 \; hrs$$

Time and Work

A can complete a work in X hours. B can complete the work in Y hours. If both work together, how many hours is required to complete the work?

Workers	Time to complete in hours	Work done in 1 hour
A	X	$\frac{1}{X}$
B	Y	$\frac{1}{Y}$
A+B		$\frac{1}{X} + \frac{1}{Y}$

In 1 hour, A completes 1/X th of the work.

In 1 hour, B completes 1/Y th of the work.

In 1 hour, if both work together, (1/X + 1/Y) th part of the work is completed

$$\frac{1}{X} + \frac{1}{Y} = \frac{X+Y}{XY}$$

Work $\frac{X+Y}{XY}$ $\xrightarrow{\text{time taken}}$ 1 hour

Work 1 (100%) $\xrightarrow{\text{time taken}}$ hours??

Total work completed is $\dfrac{XY}{X+Y}$

Example:

A needs 10 days to complete a work. B needs 15 days to complete the work. Days required to complete if both work together?

	Days	Work in 1 day	Work done together
A	10	1/10	
B	15	1/15	
A+B		1/10 + 1/15 = 1/6	6 days

$1/6^{th}$ of work done in 1 day.

Full work (1) done in $\dfrac{1\times1}{1/6}$ = 6 days

2. A can do a work in 20 days. B can do it in 24 days. C can do in 30 days. B left after 2 days. How many days do A and C require to complete?

	Days	Work in 1 day	Work done together
A	20	1/20	
B	24	1/24	
C	30	1/30	
A+B+C		1/20 + 1/24 + 1/30 = 1/8	8 days
A+C		1/20 + 1/30 = 1/12	12 days

Work by A, B, C for 2 days = $2\left(\dfrac{1}{20}+\dfrac{1}{24}+\dfrac{1}{30}\right) = 2 \times \dfrac{1}{8} = \dfrac{1}{4}$

$1/4^{th}$ of the work done in 2 days by A, B, C working together. On the 3rd day, remaining work to be done by A and C is $1 - ¼ = 3/4^{th}$ of the work.

A + C can do $1/12^{th}$ of the work in 1 day.

$$\text{Work } \frac{1}{12} \xrightarrow[\text{by A + C}]{\text{time taken}} 1 \text{ day}$$

$$\text{Work } \frac{3}{4} \xrightarrow{\text{time taken}} \text{days??}$$

$$\frac{3/4}{1/12} = 9 \; days$$

A, B, C worked for first 2 days and completed $1/4^{th}$ of the work.
A and C worked for next 9 days to complete rest $3/4^{th}$ of the work.
Total work took 11 days to complete.

3. X and Y take 18 days, Y and Z take 24 days, X and Z take 36 days to do some work. If all work together, how many days are required to complete? If all three work separately, how many days do they take?

	Days	Work in 1 day
X+Y	18	1/18
Y+Z	24	1/24
X+Z	36	1/36
2 (X+Y+Z)		1/18+1/24+1/36 = 1/8

$X + Y + Z = \dfrac{^1/_8}{2} = \dfrac{1}{16}$th of the work done in 1 day by X, Y, and Z

All three together take 16 days to complete.

	1 day combined work	1 day individual work	Days required by each
X+Y+Z	1/16		
X+Y	1/18	Z = 1/16 – 1/18 = 1/144	144 days
Y+Z	1/24	X = 1/16 – 1/24 = 1/48	48 days
X+Z	1/36	Y = 1/16 – 1/36 = 5/144	28 days

X+Y+Z = 1/16

(X+Y+Z) – (X+Z) = Y= 1/16 – 1/36 = 5/144th work

5/144th work done by Y in 1 day.

To do 100% work (1), Y needs $\dfrac{1}{^5/_{144}} = \dfrac{144}{5} \approx 28 \; days$

<u>Verification</u>
X takes 48 days
Y takes 28 days (approximately)
Z takes 144 days

In 1 day, all three together do $\dfrac{1}{48} + \dfrac{1}{28} + \dfrac{1}{144} = \dfrac{4}{63}$

4/63rd work done by all three in 1 day
100% work done in $\dfrac{1}{^4/_{63}} = \dfrac{63}{4} \approx 16$ days

4. 6 people can do some work in 8 days. How many days do 12 people need to do the same work?

Method 1:

	Days	1 day work
6 people	8	1/8th

6 people $\xrightarrow{\text{in one day, do}}$ $\dfrac{1}{8}$th work

12 people $\xrightarrow{\text{in one day, do}}$???

$$\dfrac{12 \times \dfrac{1}{8}}{6} = \dfrac{3/2}{6} = \dfrac{1}{4}$$

1/4th work is done by 12 people in 1 day. Therefore, 12 people take 4 days to do the work.

Method 2: Inverse variation unitary method

More people need less days (multiply)

6 people $\xrightarrow{\text{do work in}}$ 8 days

1 person $\xrightarrow{\text{does work in}}$ 8 x 6 days

12 people $\xrightarrow{\text{do work in}}$ $\dfrac{48 \times 1}{12}$ = 4 days

- or -
$8 \times 6 = x \times 12$. Therefore, x = 4

5. Taps A and B can fill a tank in 15 hours and 20 hours respectively. Tap C can empty the tank in 25 hours. What is the time taken for the tank to be filled if all taps are turned on?

Taps	Hours to fill/empty	1 hour work
A	15	1/15
B	20	1/20
C	25	1/25

A and B fill. C empties. Therefore, in 1 hour, work done by all three is

$$\frac{1}{15} + \frac{1}{20} - \frac{1}{25} = \frac{47}{300}$$

In 1 hour, all three fill 47/300th of the tank. Time to fill the tank is $\frac{300}{47} \approx 6.4 \; hrs$

Taps	Hours to fill/empty	1 hour work	Time to fill
A	15	1/15	
B	20	1/20	
C	25	1/25	
A+B-C		1/15+1/20-1/25=47/300	300/47 hrs