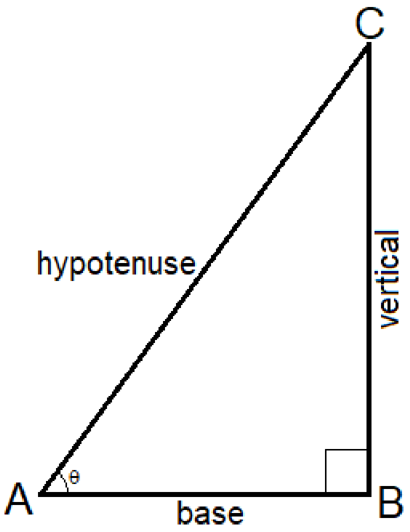
Trigonometry Basics

It is a branch of mathematics that deals with relationship between sides and angles of a right angled triangle. There are six trigonometric ratios.



The angle at vertex A is θ.
The six trigonometric ratios are:

a. $sin\theta = \frac{vert}{hyp} = \frac{BC}{AC}$

b. $cos\theta = \frac{base}{hyp} = \frac{AB}{AC}$

c. $tan\theta = \frac{vert}{base} = \frac{BC}{AB} = \frac{sin\theta}{cos\theta}$

d. $cosec\theta = \frac{hyp}{vert} = \frac{AC}{BC} = \frac{1}{sin\theta}$

e. $sec\theta = \frac{hyp}{base} = \frac{AC}{AB} = \frac{1}{cos\theta}$

f. $cot\theta = \frac{base}{vert} = \frac{AB}{BC} = \frac{1}{tan\theta}$

Trigonometric Identities
Using Pythagoras theorem, three relationships between ratios are written.

Pythagoras theorem: $base^2 + vert^2 = hyp^2$
That is, $AB^2 + BC^2 = AC^2$

1. $AB^2 + BC^2 = AC^2$
Divide both sides by AC^2

$$\frac{AB^2 + BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

$$(\frac{AB}{AC})^2 + (\frac{BC}{AC})^2 = 1$$

$(cos\theta)^2 + (sin\theta)^2 = 1$ This is written as $\boldsymbol{cos^2\theta + sin^2\theta = 1}$

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2. $AB^2 + BC^2 = AC^2$
Divide both sides by AB^2

$$\frac{AB^2 + BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

$$1 + (\frac{BC}{AB})^2 = (\frac{AC}{AB})^2$$

$1 + (tan\theta)^2 = (sec\theta)^2$ This is written as $\boldsymbol{sec^2\theta - tan^2\theta = 1}$

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3. $AB^2 + BC^2 = AC^2$
Divide both sides by BC^2

$$\frac{AB^2 + BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

$$(\frac{AB}{BC})^2 + 1 = (\frac{AC}{BC})^2$$

$(cot\theta)^2 + 1 = (cosec\theta)^2$ This is written as $\boldsymbol{cosec^2\theta - cot^2\theta = 1}$

The three trigonometric identities are:
1. $cos^2\theta + sin^2\theta = 1$
2. $sec^2\theta - tan^2\theta = 1$
3. $cosec^2\theta - cot^2\theta = 1$

Trigonometric ratios in terms of $sin\theta$
Knowing only the value of sine, we can find the other five ratios.
1. $sin\theta = sin\theta$

2. $cos\theta = \sqrt{1 - sin^2\theta}$

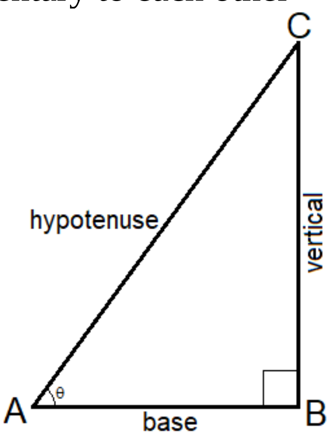
3. $tan\theta = \frac{sin\theta}{cos\theta} = \frac{sin\theta}{\sqrt{1-sin^2\theta}}$

4. $cosec\theta = \frac{1}{sin\theta}$

5. $sec\theta = \frac{1}{cos\theta} = \frac{1}{\sqrt{1-sin^2\theta}}$

6. $cot\theta = \frac{1}{tan\theta} = \frac{\sqrt{1-sin^2\theta}}{sin\theta}$

Sine and cosine are complimentary to each other



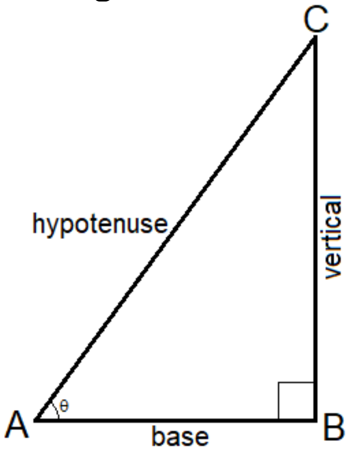
If $A = \theta$, B=90, then $C = 90 - \theta$
$$sinA = sin\theta = \frac{BC}{AC}$$

$$cosA = cos\theta = \frac{AB}{AC}$$

$sinC = \sin(90 - \theta) = \frac{AB}{AC}$ This is $cos\theta$

Therefore, $\sin(90-\theta) = cos\theta$
Similarly, $\cos(90-\theta) = sin\theta$

Trigonometric values of some angles



a. For 0^o:
When angle is 0, it means that BC = 0. Therefore, AC = AB

$$sin0 = \frac{BC}{AC} = \frac{0}{AC} = 0$$

$$cos0 = \frac{AC}{AB} = \frac{AC}{AC} = 1$$

$$tan0 = \frac{BC}{AB} = \frac{sin0}{cos0} = 0$$

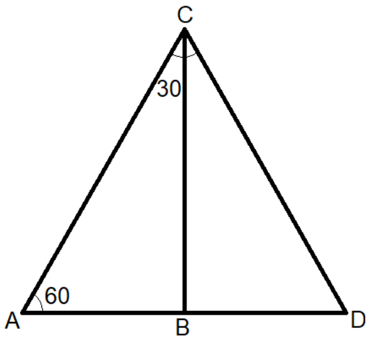
b. For 90^o:
When angle is 90, AB = 0, BC = AC

$$sin90 = \frac{BC}{AC} = \frac{AC}{AC} = 1$$

$$cos90 = \frac{AB}{AC} = \frac{0}{AC} = 0$$

$$tan90 = \frac{BC}{AB} = \frac{sin90}{cos90} = \frac{1}{0} = \infty \text{ (infinity)}$$

c. For 30^o and 60^o:

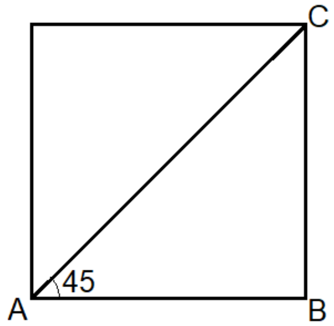


Consider an equilateral triangle ADC. Draw a vertical line from C till B.
In the triangle ABC, AC = 2AB

From Pythagoras theorem, $AB^2 + BC^2 = AC^2$
$AB^2 + BC^2 = 4AB^2$
Therefore, $BC = \sqrt{3AB^2} = AB\sqrt{3}$

$sin30 = \frac{AB}{AC} = \frac{AB}{2AB} = \frac{1}{2} = 0.5$	$sin60 = \frac{BC}{AC} = \frac{AB\sqrt{3}}{2AB} = \frac{\sqrt{3}}{2} = 0.866$
$cos30 = \sin(90 - 30) = sin60 = 0.866$	$cos60 = \sin(90 - 60) = sin30 = 0.5$
$tan30 = \frac{sin30}{cos30} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = 0.577$	$tan60 = \frac{sin60}{cos60} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} = 1.73$

d. For 45^o:



Consider a square, where diagonal AC makes angle 45^o with the sides.
AB = BC
From Pythagoras theorem, $AC = AB\sqrt{2}$

Writeach

$$sin45 = \frac{BC}{AC} = \frac{AB}{AB\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.707$$

$$cos45 = \frac{AB}{AC} = \frac{AB}{AB\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.707$$

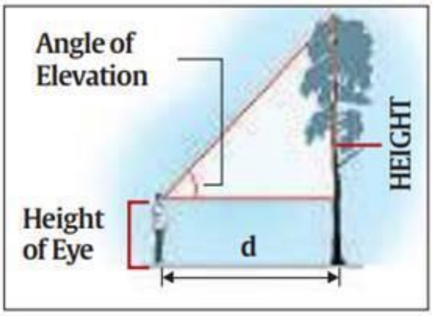
$$tan45 = \frac{BC}{AB} = \frac{AB}{AB} = 1$$

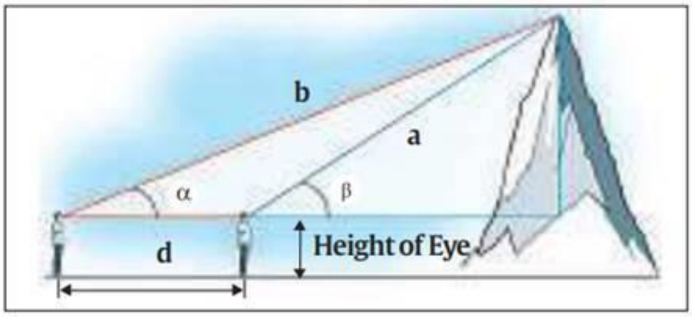
	0	30	45	60	90
sin	0	$^1/_2$	$^1/_{\sqrt{2}}$	$^{\sqrt{3}}/_2$	1
cos	1	$^{\sqrt{3}}/_2$	$^1/_{\sqrt{2}}$	$^1/_2$	0
tan	0	$^1/_{\sqrt{3}}$	1	$\sqrt{3}$	∞

Note: Values of sine and cosine can never be more than 1.

Applications of Trigonometry

a. It is used to find the heights of tall structures such as buildings and trees.
b. The heights of Himalayan mountains such as Everest were calculated using trigonometric ratios.





c. It is used to find the width of rivers.
d. Land areas such as states and countries are calculated using trigonometry.
e. Trigonometric ratios are extensively used in physics and engineering.
f. AC (alternating current) waves are called sine waves as they follow a path given by the sine ratio.