Simple Interest and Compound Interest

Simple Interest

a. One time deposit, calculated annually:

$$SI = \frac{PTr}{100}$$

P – principal, or sum deposited
r – rate of interest
T – number of years

Amount after T years, A = P + SI

b. Monthly investment:

$$SI = P \times \frac{n(n+1)}{2} \times \frac{r}{12 \times 100}$$

P – principal, or sum deposited every month
R – rate of interest per month (R = r/12)
n – number of months

For n months, total deposit = Pn
Maturity value after n months = Pn + SI

Compound Interest

For 1 year, simple interest and compound interest are equal.

* Without using formula:
SI calculated for 1st year. The amount of 1st year is the principal for the 2nd year. The amount of each year is the principal for the next year.

$A_1 = P_1 + \frac{P_1 r}{100}$ $P_2 = A_1$	$A_2 = P_2 + \frac{P_2 r}{100}$ $P_3 = A_2$	$A_3 = P_3 + \frac{P_3 r}{100}$ $P_4 = A_3$

…and so on

* Using formula:

$$A = P(1 + \frac{r}{100})^n$$

CI = A – P

A - amount
P – principal deposited
r – rate of interest
n – number of years

* Fractional compounding:

$$A = P(1 + \frac{r/x}{100})^{xn}$$

x = 2 for half yearly compounding
x = 4 for quarterly compounding

* If time is in months less than a year:

$$A = P(1 + \frac{r/x}{100})^{xm/12}$$

m – number of months

* If rate of interest is different for different years:

$$A = P\left(1 + \frac{r_1}{100}\right)^x (1 + \frac{r_2}{100})^y$$

r_1 – rate of interest for x years
r_2 – rate of interest for y years

Depreciation

$$A = P(1 - \frac{r}{100})^n$$

Simple Interest Examples:

1. For a principal of Rs.8100, find the amount after 3 years, 4 months, at a rate of simple interest 8 1/3%

P = Rs.8100
r = 25/3 %
T = 3 4/12 = 3 1/3 years or 10/3 years

$$SI = \frac{PTr}{100} = \frac{8100 \times \frac{10}{3} \times \frac{25}{3}}{100} = 2250$$

Simple interest is Rs.2250
Amount after 3 y, 4 m is A = P + SI = 8100 + 2250 = Rs.10350

2. Find the simple interest and amount on Rs.9125 for the period from April 14 to October 20, at a rate of 8%

P = 9125
r = 8%

April 14th to 30th: 17 days
May: 31 days
June: 30 days
July: 31 days
Aug: 31 days
Sept: 30 days
Oct 1st to 20th: 20 days
Total days: 190 days

T = 190/365

$$SI = \frac{PTr}{100} = \frac{9125 \times \frac{190}{365} \times 8}{100} = 380$$

Simple interest from April 14th to October 20th is Rs.380
Amount for 190 days is A = P + SI = 9125 + 380 = Rs.9505

3. If the simple rate of interest is 10%, in how many years the amount received will be double the amount deposited?

$$A = P + \frac{PTr}{100}$$

A = 2P

$2P = P + \frac{PTr}{100}$

Solving, $Tr = 100$

r = 10%

T = 100/10 = 10

At a rate of 10%, the amount will double in 10 years.

Compound Interest Examples:

1. Find the compound interest on Rs.20000 at rate of interest 12% for 2 years

a. compounded annually

b. compounded half yearly

c. compounded quarterly

P = 20000

r = 12%

n = 2 years

$A = P(1 + \frac{r/x}{100})^{xn}$

Compounding	**x**	**Amount**	**CI = A - P**
Annually	1	$A = 20000(1 + \frac{12}{100})^2 = 25088$	Rs.5088
Half yearly	2	$A = 20000(1 + \frac{12/2}{100})^{2\times2} \approx 25249$	Rs.5249
Quarterly	4	$A = 20000(1 + \frac{12/4}{100})^{4\times2} \approx 25335$	Rs.5335

Verification of half yearly compounding using simple interest
There are 4 half years in 2 years. Calculated 4 times.

$$A_1 = P_1 + \frac{P_1 r}{100} = 20000 + \frac{20000 \times 12 \times \frac{1}{2}}{100} = 21200$$
$$P_2 = A_1 = 21200$$

$$A_2 = P_2 + \frac{P_2 r}{100} = 21200 + \frac{21200 \times 12 \times \frac{1}{2}}{100} = 22472$$
$$P_3 = A_2 = 22472$$

$$A_3 = P_3 + \frac{P_3 r}{100} = 22472 + \frac{22472 \times 12 \times \frac{1}{2}}{100} \approx 23820$$
$$P_4 = A_3 = 23820$$

$$A_4 = P_4 + \frac{P_4 r}{100} = 23820 + \frac{23820 \times 12 \times \frac{1}{2}}{100} \approx 25249$$

For quarterly compounding, the SI equation to be multiplied by ¼, and calculated 8 times (8 quarters in 2 years)

2. Find the compound interest on Rs.12000 at rate of interest 8% for 1 ½ years
a. compounded annually
b. compounded half yearly
c. compounded quarterly

P = 12000
r = 8%
n = 1 ½ years = 3/2 years

$$A = P(1 + \frac{r/x}{100})^{xn}$$

Compounding	x	Amount	CI = A - P
Annually	1	$A = 12000(1 + \frac{8}{100})^{3/2} \approx 13468$	Rs.1468
Half yearly	2	$A = 12000(1 + \frac{8/2}{100})^{2\times 3/2} \approx 13498$	Rs.1498
Quarterly	4	$A = 12000(1 + \frac{8/4}{100})^{4\times 3/2} \approx 13514$	Rs.1514

3. Find the compound interest on Rs.15000 at rate of interest 10% for 2 years, and 8% for the next year.

$$A = P\left(1 + \frac{r_1}{100}\right)^x (1 + \frac{r_2}{100})^y$$

P = 15000
r_1 = 10% for x = 2 years
r_2 = 8% for y = 1 year

$$A = 15000\left(1 + \frac{10}{100}\right)^2 \left(1 + \frac{8}{10}\right) = 19602$$

CI = A – P = Rs. 4602

Verification using simple interest

$$A_1 = P_1 + \frac{P_1 r_1}{100} = 15000 + \frac{15000 \times 10}{100} = 16500$$
$$P_2 = A_1 = 16500$$

$$A_2 = P_2 + \frac{P_2 r_1}{100} = 16500 + \frac{16500 \times 10}{100} = 18150$$
$$P_3 = A_2 = 18150$$

$$A_3 = P_3 + \frac{P_3 r_2}{100} = 18150 + \frac{18150 \times 8}{100} = 19602$$