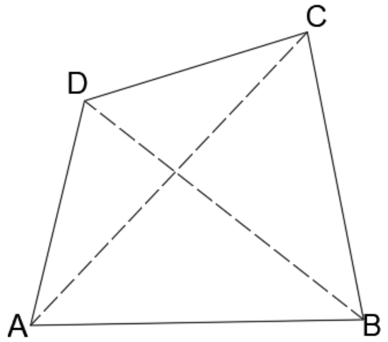
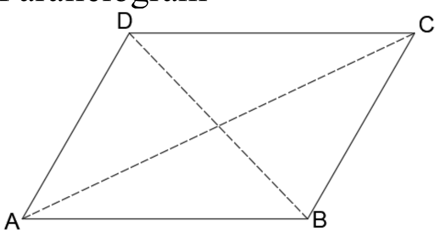
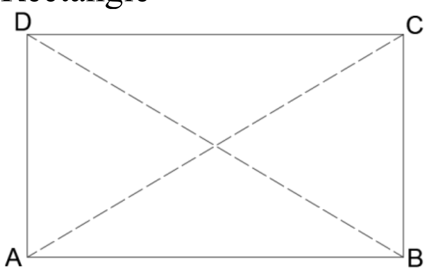
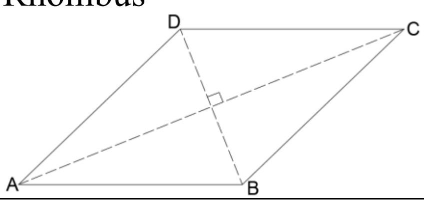
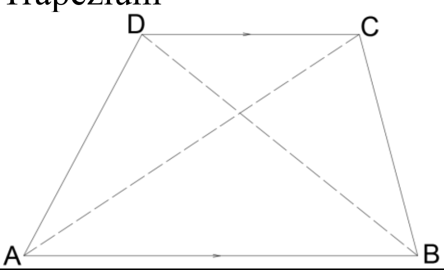
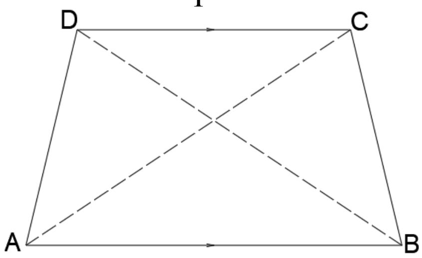
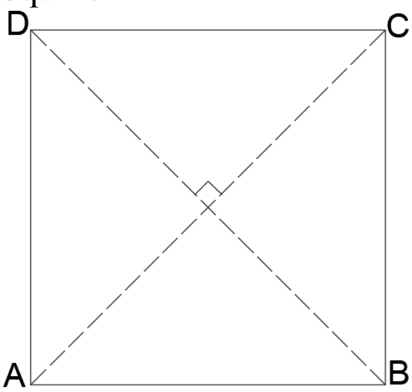
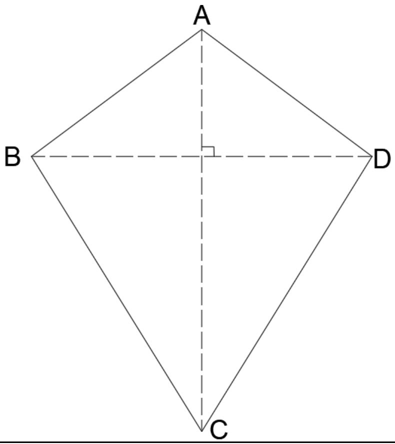
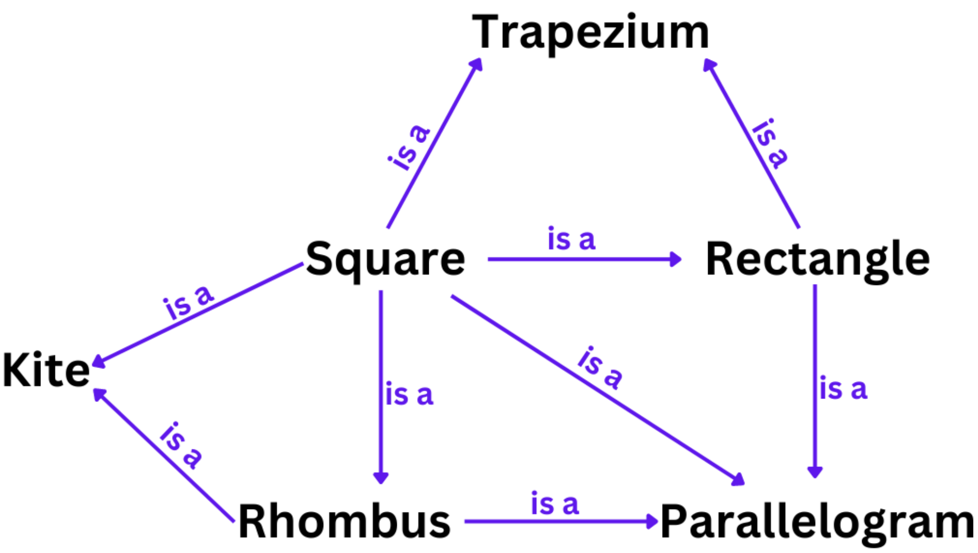
Quadrilaterals

Quadrilaterals are polygons with 4 sides, 4 angles, 4 vertices, 2 diagonals. The sum of the angles of a quadrilateral is 360°.

Quadrilateral	**Sides**	**Angles**	**Diagonals**
Quadrilateral 	* Unequal * Not parallel	Unequal	Unequal
Parallelogram 	* Opposite sides equal * Opposite sides parallel	Opposite angles equal	* Unequal * Bisect each other
Rectangle 	* Opposite sides equal * Opposite sides parallel	All angles equal to 90°	* Equal * Bisect each other
Rhombus 	* All sides equal *Opposite sides parallel	Opposite angles equal (A=C, B=D)	* Unequal * Bisect each other * Intersect at 90°
Trapezium 	* One pair of opposite sides parallel (AB‖CD)	Unequal	Unequal

Isosceles Trapezium 	* One pair of opposite sides parallel * Non-parallel sides equal	Pair of adjacent angles equal (A=B, C=D)	Equal
Square 	* All sides equal * Opposite sides parallel	All angles equal to 90°	* Equal * Bisect each other at 90°
Kite 	One pair of adjacent sides equal	One pair of opposite angles equal	* Unequal * Intersect at 90° * One diagonal is bisected by the other



Properties

Parallelogram:

1. The diagonal of a parallelogram divides it into two congruent triangles.
2. Opposite angles are equal.
3. Pair of opposite sides are equal and parallel to each other.
4. Diagonals bisect each other.
5. Each diagonal divide the parallelogram into two congruent triangles.

Rhombus:

1. Rhombus is a parallelogram with all sides equal.
2. Diagonals bisect each other at right angles.

Rectangle:

1. Rectangle is a parallelogram.
2. Diagonals are equal.
3. All angles are 90^{o}.

Trapezium:

1. One pair of opposite sides are parallel.
2. In an isosceles trapezium, diagonals are equal.
3. In an isosceles trapezium, non-parallel sides are equal.

Kite:

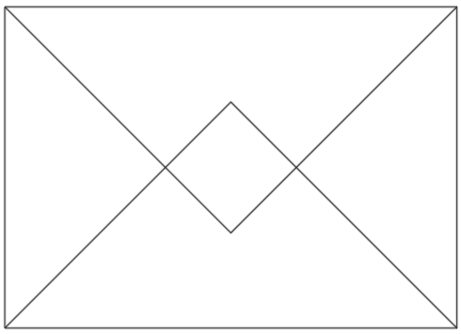
1. Pair of adjacent sides are equal.
2. One diagonal is bisected by the other.
3. Diagonals intersect at right angles.

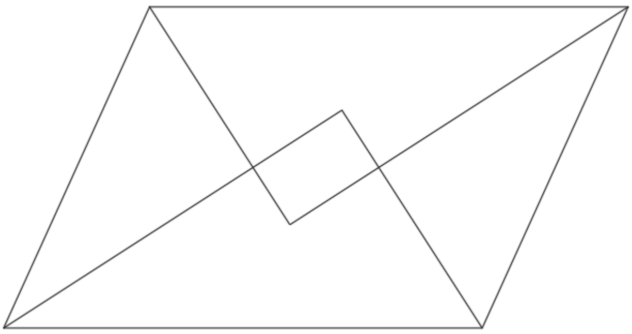
Square:

1. Square has all the properties of a parallelogram, rectangle, rhombus, trapezium, or kite.

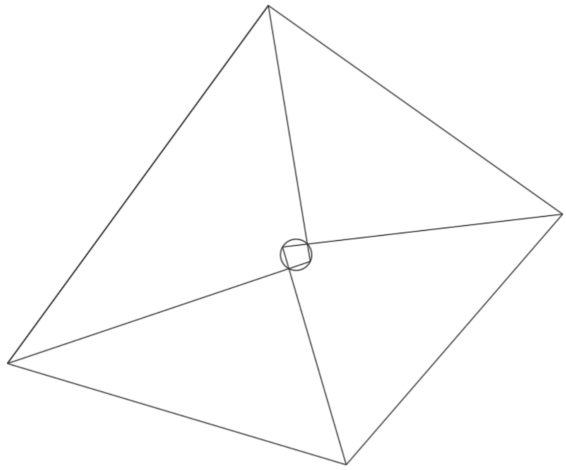
Angle bisectors of quadrilaterals

All angle bisectors of rectangle form a square
All angle bisectors of parallelogram form a rectangle

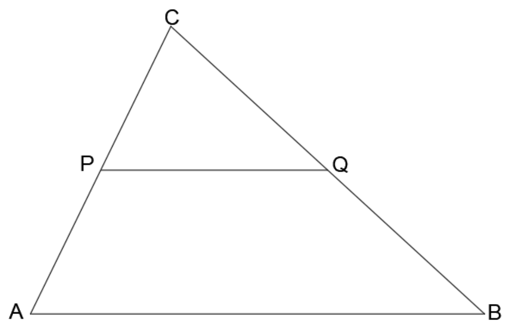




The angle bisectors of a quadrilateral always form a cyclic quadrilateral



Midpoint Theorem

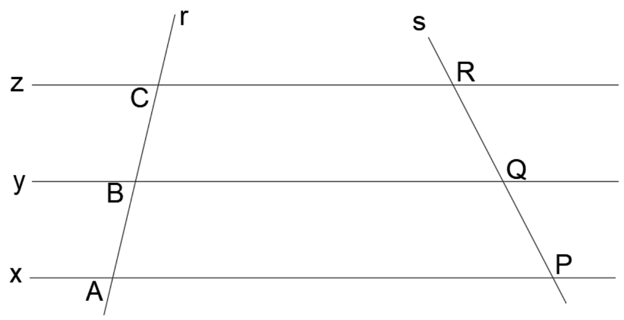


The line segments joining the *midpoints* of two sides of a triangle are *parallel* to the third side. The line is also half the third side.
PQ||AB and PQ = ½ AB

Converse of midpoint theorem:
A line passing through the midpoint of one side of a triangle and *parallel* to the third side will pass through the *midpoint* of the second side.

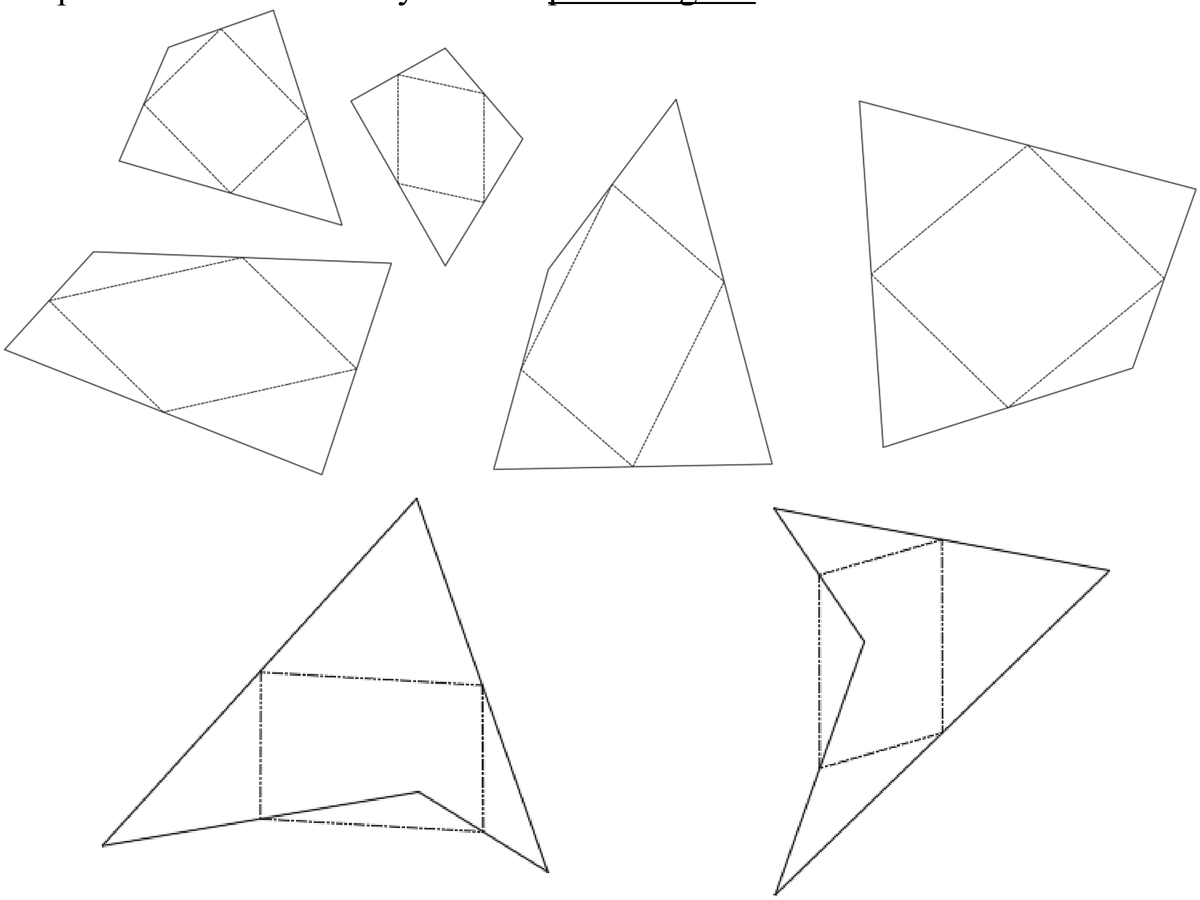
If a line is passing through P and is parallel to AB, then Q is the midpoint of BC.

Intercept theorem



Three parallel lines x, y, z are equidistant. r is a transversal and its intercepts are A, B, C. If AB = BC, then in any other transversal s, PQ = QR.

* Irrespective of the shape of the irregular quadrilateral, the lines joining the midpoints of its sides always form a parallelogram.



* The lines joining the midpoints of –
- a rectangle form a rhombus
- a rhombus form a rectangle
- a parallelogram form a parallelogram
- a trapezium form a parallelogram
- a kite form a rectangle
- a square form a square

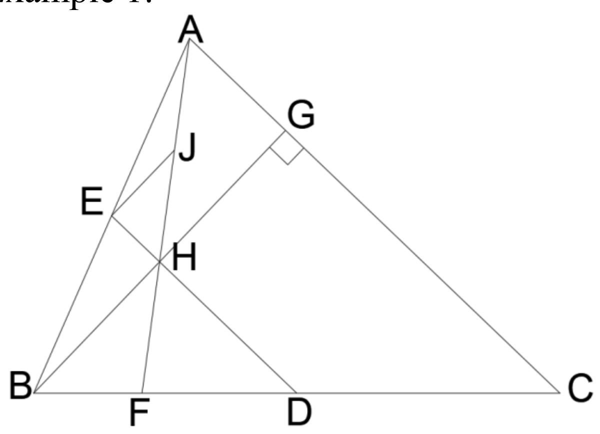
* In a quadrilateral that has diagonals intersecting at **right angles** (square, rhombus, kite), the midpoint quadrilateral is either a **rectangle** or a **square**.

The properties that help solve questions related to quadrilaterals are:

- **Parallel line property**:
  - Alternate interior angles are equal
  - Adjacent angles add to 180

    - Opposite angles are equal
    - Corresponding angles are equal
- **Congruency**: SSS, SAS, ASA, AAS, RHS
- **Midpoint theorem** or its **converse**
    - Line joining midpoint to midpoint is parallel to and half of third side
    - Line passing through one midpoint and parallel to another side
- **Intercept theorem** (Only if three or more parallel lines are present)
    - Lines crossing the parallel lines as transversals

Example 1:



D is midpoint of BC.
E is midpoint of AB.
BG is perpendicular to AC.
AF passes through intersection of DE and BG at H.
J is midpoint of AH

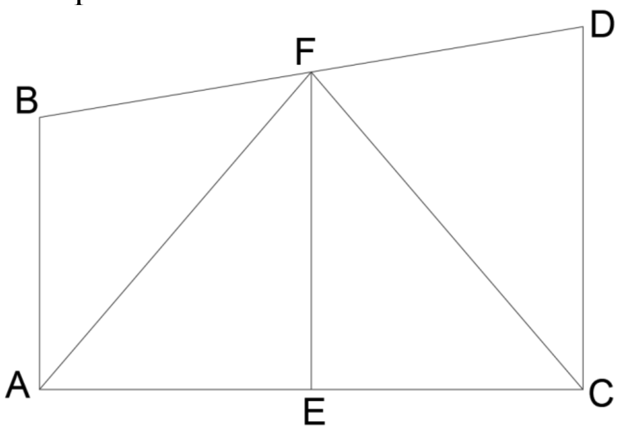
Prove that JEH is 90°
If angle AHB = 150°, find angle EHJ

a. Since there are midpoints of lines, we must use **midpoint theorem**.

In triangle ABC, ED || AC
In triangle ABH, JE || BH or JE || BG
BG is perpendicular to AC. Therefore, JE is perpendicular to ED.
So, JEH=90°

b. Given, $\angle AHB = 150°$
EJ || BH. Therefore, AJE and AHB are **corresponding angles**.
Therefore, $\angle AJE = 150°$
EJH = 180 – 150 = 30°
EHJ = 90 – 30 = 60°

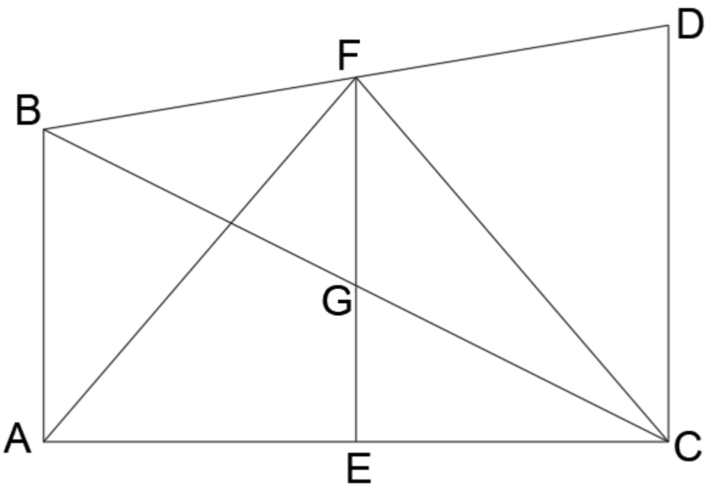
Example 2:



AB, EF, CD are parallel lines, perpendicular to AC.
AE = EC
AB = 30 cm
CD = 40 cm

Find EF
Prove that AF = FC

Construction required as the given information is not enough to solve.
a. Join BC to create triangles. It crosses EF at G



b. Given: AB || EF || CD, and AE = EC. AE and BD are transversal to the three parallel lines.
Using **intercept theorem**, BF = FD.
E is midpoint of AC
F is midpoint of BD
From intercept theorem, G is midpoint of BC, as BC is also a transversal to the three parallel lines

c. In triangle BCD, G and F are midpoints of BC and BD respectively.
According to **midpoint theorem**, FG = ½ CD. Therefore, FG = ½ 40 = 20 cm

d. In triangle ABC, G and E are midpoints of BC and AC respectively.
According to midpoint theorem, GE = ½ AB = ½ 30 = 15 cm

EF = GE + FG = 35 cm

d. In triangles AEF and CEF –

AE = EC	(Side)
$\angle AEF = \angle CEF = 90°$	(Angle)
EF is common	(Side)

Using **SAS congruency** principle, $\triangle AEF \cong \triangle CEF$
Therefore, AF = FC