Polynomials

Expression(s)	Term
0	Zero polynomial
x	Monomials
x^2	
4	
$-5x$	
$2x^5$	
$x + 2$	Binomials
$2y - 6$	
$a^3 - 4$	

Examples	Linear	Quadratic	Cubic
Monomial	$2x$	x^2	$4x^3$
Binomial	$3x + 8$	$x^2 - 25$	$8a^3 + 27b^3$
Trinomial		$3x^2 - 5x + 4$	$3x^3 + 5y^3 - 15xy$

An expression cannot be called a polynomial if any of the terms contain a power **less than 0**.
$\sqrt{x}$, $a^{0.25}$, $\frac{1}{x}$ are not polynomials.

	Constant	Variable(s)	Exponent
$5x$	5	x	1
$-4x^2$	-4	x	2
$24ab$	24	ab	1
9	9	x	0
$\frac{3}{5}ab^2$	$\frac{3}{5}$	ab	2
$\frac{1}{x^2}$	1	x	-2

Zeroes (Roots) of a polynomial:
Any value(s) of a variable that makes the polynomial equal to zero.

$x - 1$
When x = 1, the expression becomes zero.

$a^2 = 49$
Roots are a = +7 and a = -7

$2x + 7$
The root is -7/2

Linear expressions have **one** root
Quadratic expressions have **two** roots
Cubic expressions have **three** roots

Remainder theorem
To find the remainder when one polynomial is divided by another, without long division.

Example:
Find remainder when $x^3 - 3x^2 + 5x - 1$ is divided by $x + 2$

1. Find the zero (root) of the divisor.
$x + 2 = 0.$ Therefore $x = -2$

2. Substitute this root in the dividend $x^3 - 3x^2 + 5x - 1$
$p(-2) = (-2)^3 - 3(-2)^2 + 5(-2) - 1$
$p(-2) = -31$
The remainder is -31

Factorisation
It means finding the factors of a polynomial.
A quadratic polynomial has two factors.
A cubic polynomial has three factors.

Finding factors using Factor theorem (for both quadratic and cubic)
A. Find factors of $x^2 + 3x - 10$

1. This is in the form of $px^2 + qx + r$
Find the factors of pr
Factors of 10 are +1, +2, +5, +10, -1, -2, -5, -10

2. Substitute one by one till the expression equates to zero.
$p(1) = 1^2 + 3(1) - 10$ This is not equal to zero
$p(2) = 2^2 + 3(2) - 10$ This is equal to zero
2 is one of the roots, and $x - 2$ is one of the factors

3. The second factor can be found either by long division, or by substituting the other factors.

$$p(-5) = (-5)^2 + 3(-5) - 10 = 0$$

4. The factors are $(x-2)(x+5)$ and the roots are $x=+2, x=-5$

B. Find factors of a^3-2a^2-5a+6

1. Factors of 6 are +1, -1, +2, -2, +3, -3, +6, -6

2. $p(1)=1^3-2(1)^2-5(1)+6=0$ $\quad a-1$ is one of the factors
   $p(3)=3^3-2(3)^2-5(3)+6=0$ $\quad a-3$ is one of the factors
   $p(-2)=(-2)^3-2(-2)^2-5(-2)+6=0$ $\quad a+2$ is one of the factors

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C. Find factors of $2x^2+5x-3$ using Factor theorem

1. When the coefficient of the highest power is more than 1, make it 1
$2(x^2+\frac{5}{2}x-\frac{3}{2})$

2. Find the factors of 3/2
+1, -1, +3/2, -3/2, +1/2, -1/2, +3, -3 are some of the factors

3. Substituting one by one, the factors are $(2x-1)(x+3)$
Roots are $x={}^1/_2$ and -3

Finding factors using Splitting the Middle Term theorem (only quadratic)

A. Factorise x^2-5x+6

1. The equation is of the form px^2+qx+r
Find the factors of pr
pr = 6
Factors of 6 are 1, 2, 3, 6

2. When pr is **positive**, the factors must be **added** to form the middle term, q
   When pr is **negative**, the factors must be **subtracted** to form q
$pr=+6$
$q=-5$

2 + 3 = 5. So, 2 and 3 must be selected such that their addition gives -5
-2 – 3 = -5

3. x^2-5x+6 is changed to $x^2(-2x-3x)+6$
$x^2-2x-3x+6$

4. Take common out of 1st and 2nd, and 3rd and 4th
$x(x-2)-3(x-2)$
$(x-2)(x-3)$ are the factors. Roots are 2 and 3

B. Factorize $2a^2-5a-3$

1. $2 \times 3 = 6$
2. Factors of 6 are 1, 2, 3, 6
3. pr = -6. Since it is negative, factors must be subtracted to get the middle term, -5
4. 1 – 6 = -5
5. $2a^2-a+6a-3$
6. $a(2a-1)+3(2a-1)$
   $(2a-1)(a+3)$ are the factors. Roots are ½ and -3

Algebraic Identities

I. $(x+y)^2=x^2+2xy+y^2$
II. $(x-y)^2=x^2-2xy+y^2$
III. $x^2-y^2=(x+y)(x-y)$
IV. $(x+a)(x+b)=x^2+(a+b)x+ab$

A. Find 98×102
$(100-2)(100+2)=100^2-2^2=10000-4=9996$

B. Factorise $36x^2-25y^2$
$(6x)^2-(5y)^2=(6x+5y)(6x-5y)$

C. Factorise $9x^2+12xy+4y^2$
1. $(3x)^2+2(3x)(2y)+(2y)^2$
2. It is in the form $a^2+2ab+b^2=(a+b)^2$
3. Factors are $(3x+2y)^2$

V. $(x+y)^3=x^3+y^3+3xy(x+y)$
VI. $(x-y)^3=x^3-y^3-3xy(x-y)$
VII. If $x+y+z=0$, $x^3+y^3+z^3=3xyz$

Factorise quadratic equation by splitting the middle term