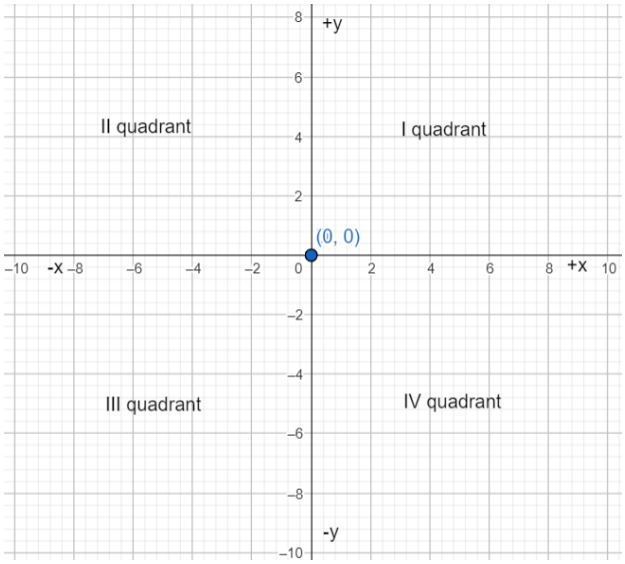
Coordinate Geometry

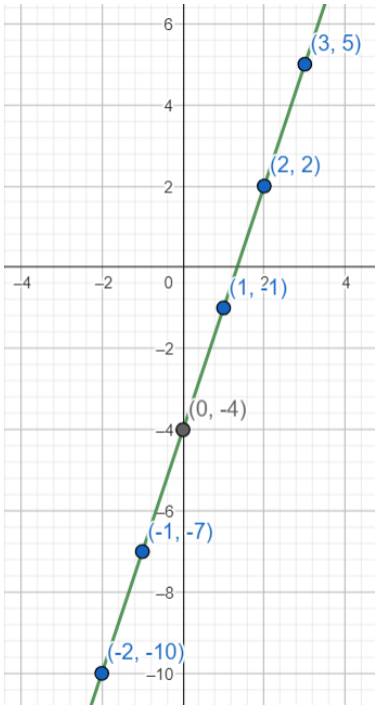
Cartesian plane has two axes on a plane surface. The horizontal axis is the x-axis, the vertical axis is the y-axis. The intersection of x and y axes is called the origin. Any point in the cartesian plane has an x-coordinate (abscissa) and y-coordinate (ordinate).



1. Draw the graph of the equation $3x - y = 4$

Rewrite the equation in terms of y.
$y = 3x - 4$
x is called independent variable; y is called dependent variable.

x	y
0	-4
1	-1
2	2
3	5
-1	-7
-2	-10



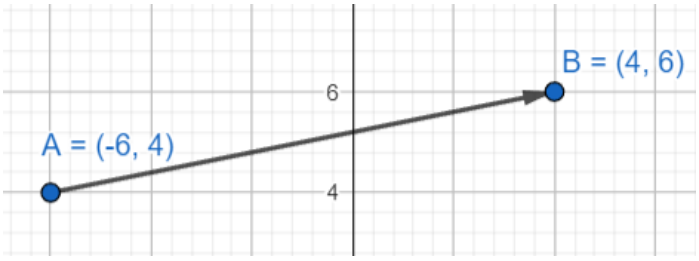
Plotting these values on the graph, a straight line is formed. Therefore, equations having maximum power as 1 are called linear equations.

Distance Formula

It is the formula for finding the distance between two points on a line in the cartesian plane.

$$A(x_1, y_1)$$
$$B(x_2, y_2)$$

Length of the line AB is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

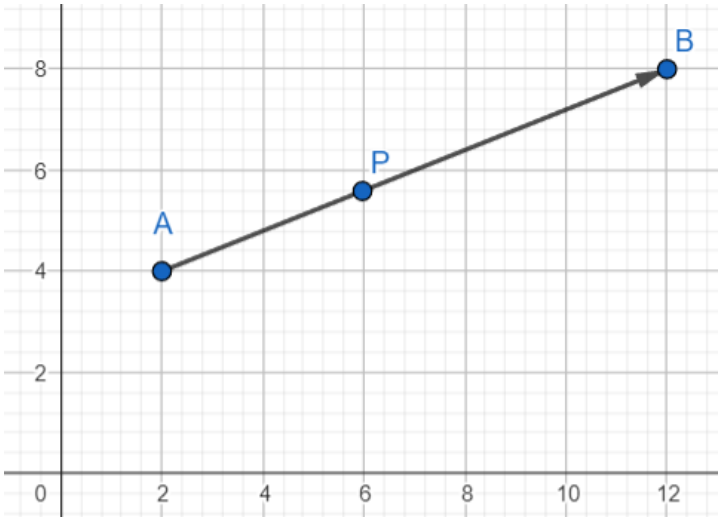


A (-6, 4)
B (4, 6)

$$AB = \sqrt{(4 - (-6))^2 + (6 - 4)^2} = \sqrt{104}$$

Section Formula

It is used to find the coordinates of a point that divides a line into a given ratio.



Line AB is between two points $A\ (x_1, y_1)$ and $B(x_2, y_2)$
Point P divides the line in the ratio m_1 and m_2
Section formula is used to find the coordinates of point P.

$$P_x = \frac{m_1x_2+m_2x_1}{m_1+m_2} \qquad P_y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

Example:
Find the coordinates of the point P that divides a line with A (2, 4) and B (12, 8) in the ratio 2:3

	x	y
A	2	4
B	12	8

$m_1 = 2$, $m_2 = 3$

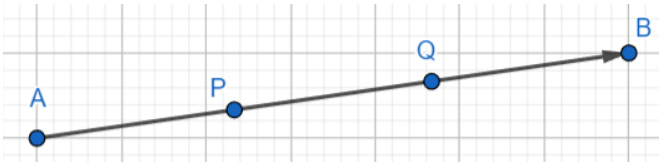
$$P_x = \frac{2 \times 12 + 3 \times 2}{2 + 3} = 6$$

$$P_y = \frac{2 \times 8 + 3 \times 4}{2 + 3} = {}^{28}/_{5}$$

Midpoint Formula
When the point P is in the middle of the line AB, the ratio is 1:1

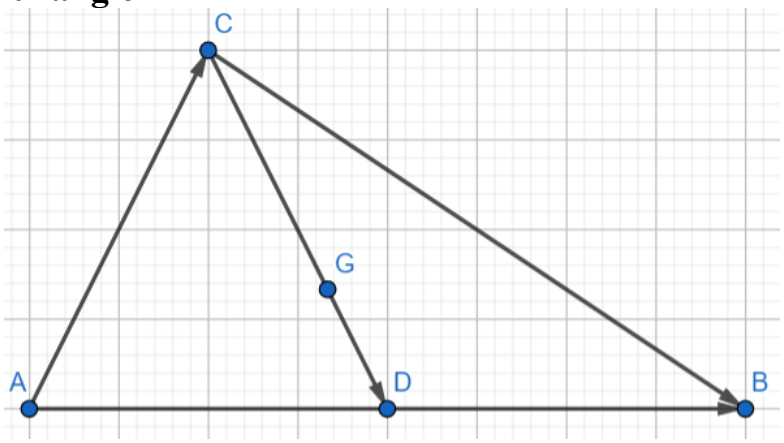
$$P_x = \frac{x_1+x_2}{2} \qquad P_y = \frac{y_1+y_2}{2}$$

Points of Trisection



P and Q divide the line AB into three equal parts. The section formula is used to find the coordinates of P and Q.
For point P, $m_1:m_2 = 1:2$
For point Q, $m_1:m_2 = 2:1$

Centroid of a triangle



The line joining the vertex of a triangle to the midpoint of its opposite side is called the median. AD is one of the medians of this triangle, as D is the midpoint of AB. All the three medians of a triangle meet at a single point called the centroid. Here, G is the centroid.

In any triangle, the centroid point is at a ratio of 2:1 in the median. So, CG = 2GD.

Method 1:
a. Using the midpoint formula, find the coordinates of point D.
b. Using the section formula, with ratio 2:1 from A to D, find the coordinates of the centroid, G

Method 2: Direct method
$A\ (x_1, y_1),\quad B\ (x_2, y_2),\quad C\ (x_3, y_3)$

Centroid coordinates are $G\left(\frac{x_1+x_2+x_3}{3}, \frac{y+y_2+y_3}{3}\right)$

Equation of a line
A linear equation is in the form $ax + by = z$

Inclination of a line is the angle a line makes with the x-axis. It is always measured in anticlockwise direction.

Slope or **Gradient**
If θ is the inclination angle of a line with the x-axis, then slope or gradient of the line is $tan\theta$.

$$slope = \tan(inclination)$$
$$m = tan\theta$$

The equation of a line can be written in terms of y.

$$ax + by = z$$

$$by = -ax + z$$

$$y = -\frac{a}{b}x + \frac{z}{a}$$

$$Slope\ m = -\frac{a}{b},\ \ y - intercept\ c = \frac{z}{a}$$

y-intercept is the point where the line crosses the y-axis.
The equation of a line in terms of slope and y-intercept is $y = mx + c$

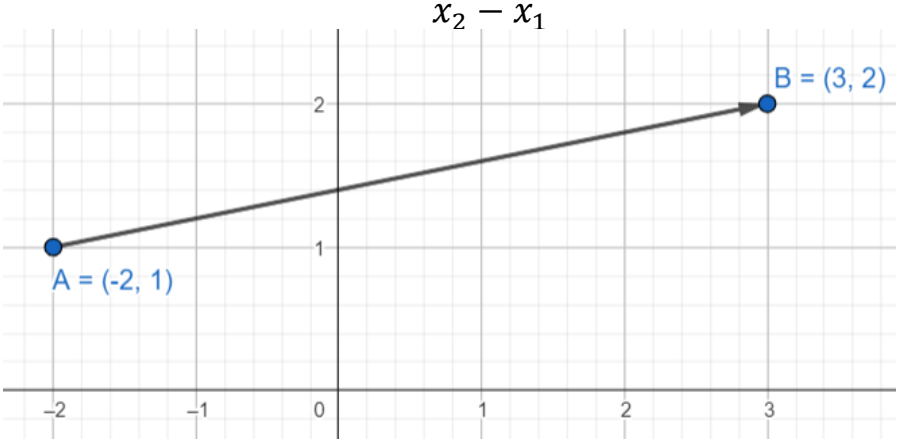
Example:

$$6x - 3y = 2$$
$$y = 2x - {}^{2}/_{3}$$

Slope of the line is +2, and it crosses the y-axis at -2/3
Positive slope means the line rises from left to right.
Negative slope means the line descends from left to right.

Two points form: The slope of a line when the end coordinates are known.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



A (-2, 1), B (3, 2)

$$m = \frac{2 - 1}{3 - (-2)} = \frac{1}{5}$$

Slope the ratio of the vertical height to the horizontal distance between the two end points of the line. The vertical rise is 1 unit, the horizontal distance is 5 units. Therefore, slope is 1/5.

Point-slope form: The slope of a line when one point on the line is known.

$$m = \frac{y - y_1}{x - x_1}$$

This gives the equation of the line when slope and one point on it are known.

Slope-intercept form: The slope of a line when its y-intercept is known.

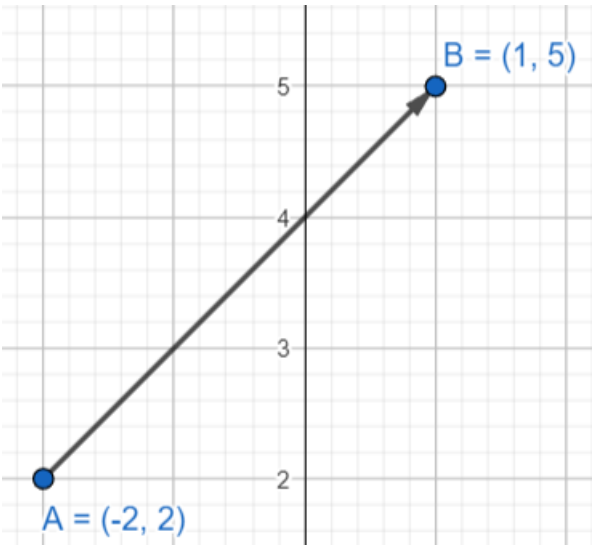
$$y = mx + c$$
$$m = \frac{y - c}{x}$$

Combining all the three,

$$\boldsymbol{m = \frac{y - c}{x} = \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}}$$

Using any of the three, the equation of a line can be found.

Example:



A (-2, 2)
B (1, 5)

$$m = \frac{5 - 2}{1 - (-2)} = \frac{3}{3} = 1$$

The equation of the line can be found in two ways:

1. $m = \frac{y - y_1}{x - x_1}$. Here, any one of the two points can be used. Taking B coordinates of x=1, y=5,

$$1 = \frac{y-5}{x-1}$$

$$x - 1 = y - 5$$
$$x - y = -4$$

2. $y = mx + c$. Taking B coordinates, x=1, y=5,

$$5 = 1 \times 1 + c$$

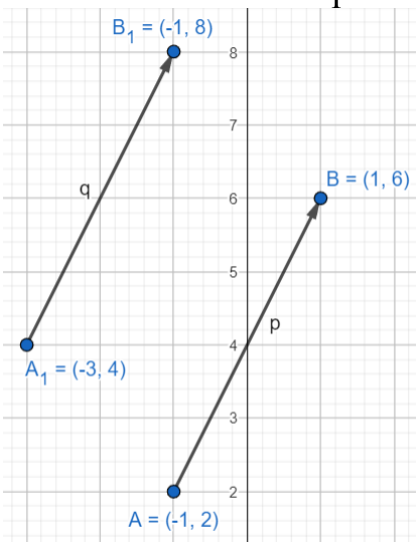
Therefore, $c = 4$

Use this value of c in the same equation:

$$y = x + 4$$
$$x - y = -4$$

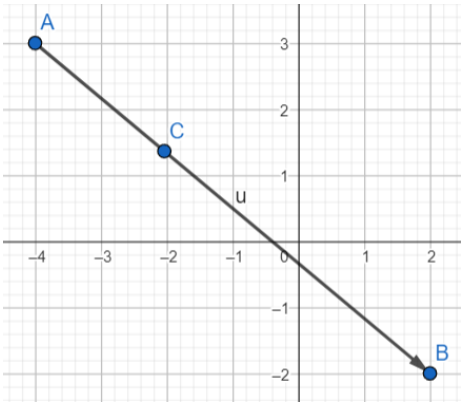
Taking A coordinates of x=-2, y=2 also gives the same equation.

All lines parallel to each other have the same slope.

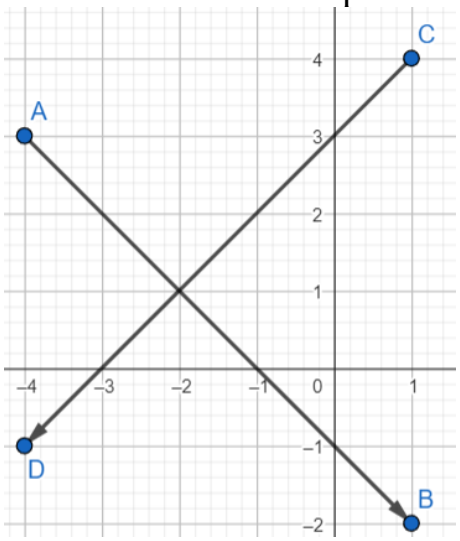


Line p and q are parallel to each other. Both have the same slope of +2.

Three or more points are collinear (lie on the same line) if all the sections of the line have the same slope. Sections AC, CB, and AB have the same slope. So, A, C, and B are collinear.

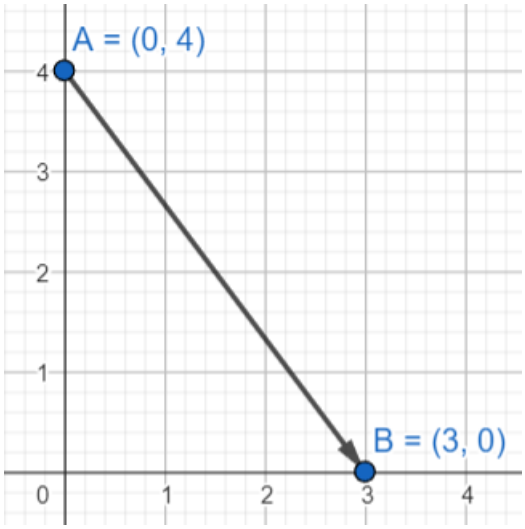


Two lines are perpendicular to each other if the product of their slopes is -1.



Slope of line AB is m_1. Slope of CD is m_2. If they are perpendicular to each other, then $m_1 m_2 = -1$

If a line has its end points on the two axes, then their coordinates will be A (0, y) and B (x, 0)



Point of intersection of two lines

The point of intersection of two lines is got by solving the two simultaneous linear equation. This point is common to both lines.

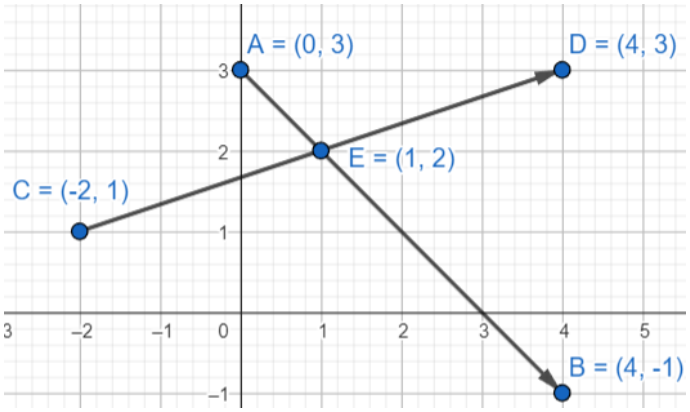
Example:

Two lines are represented by their equations:

AB: $x + y = 3$

CD: $x - 3y = -5$

Their intersecting point is on both lines. Solving these two simultaneous equations, we get x = 1, y = 2



Note:
Slope of a horizontal line is 0.
Slope of a vertical line is infinity.