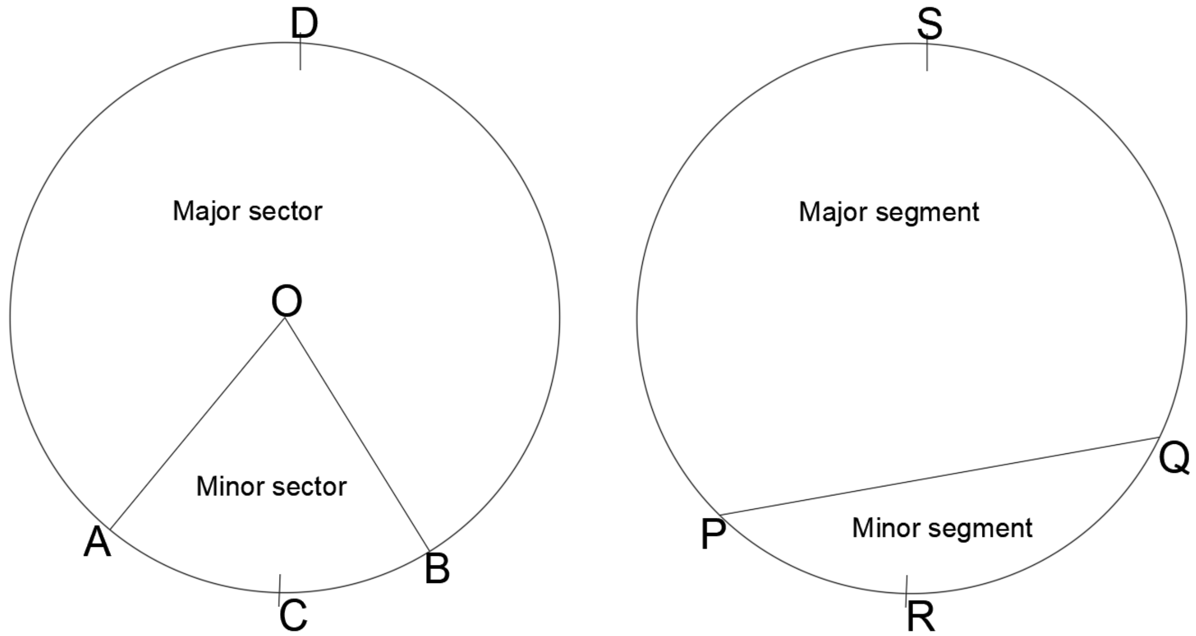
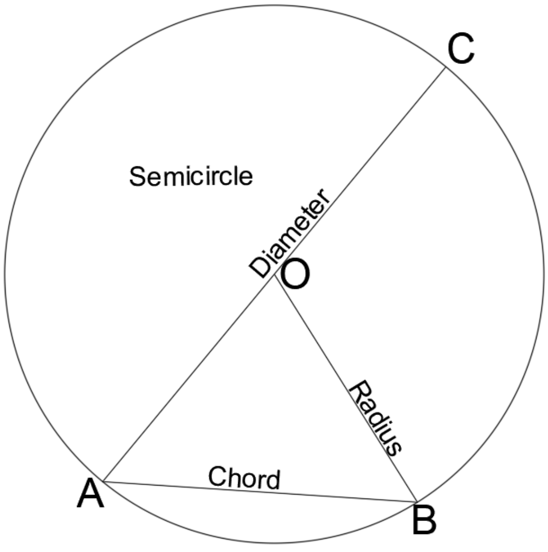
Circles



Sector: Part of a circle between two radii and an arc
Minor sector: AOBC
Major sector: ADBO

Segment: Part of a circle between a chord and an arc
Minor segment: PQR
Major segment: PQS

Arc: The curved part of the circumference
Minor arc: ACB and PRQ
Major arc: ADB and PSQ



Radius: Line joining the centre to any point on the circumference
OA, OB. OC are radii

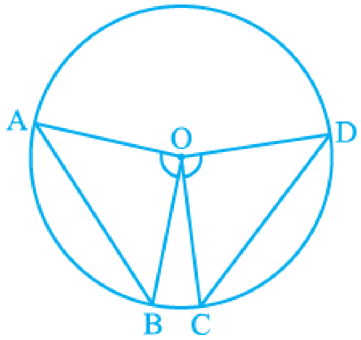
Chord: Line joining two points on the circumference
AB is chord
Diameter is the longest chord in a circle. AC is diameter

Semicircle: Half of a circle between the diameter and the arc joining its ends

Area of a circle: πr^2
Circumference of a circle: $2\pi r$

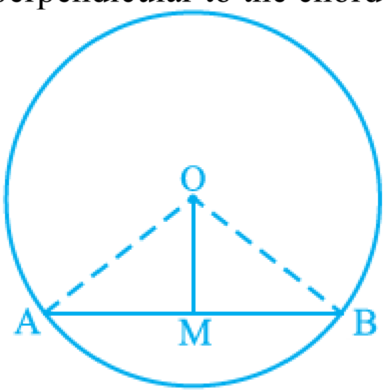
$$\pi = \frac{circumference}{diameter} = 3.14\ (or\ roughly\ \frac{22}{7})$$

<u>Theorems</u>
Theorem 1: Equal chords of a circle subtend equal angles at the centre
Theorem 2: If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal



1. If AB = CD, then $\angle AOB = \angle COD$
2. If $\angle AOB = \angle COD$, then AB = CD

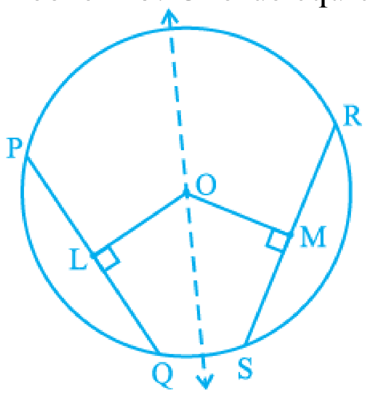
Theorem 3: The perpendicular from the centre of a circle to a chord bisects the chord.
Theorem 4: The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord



3. OM is perpendicular to AB. Then AM = MB
4. If AM = MB, then OM is perpendicular to AB

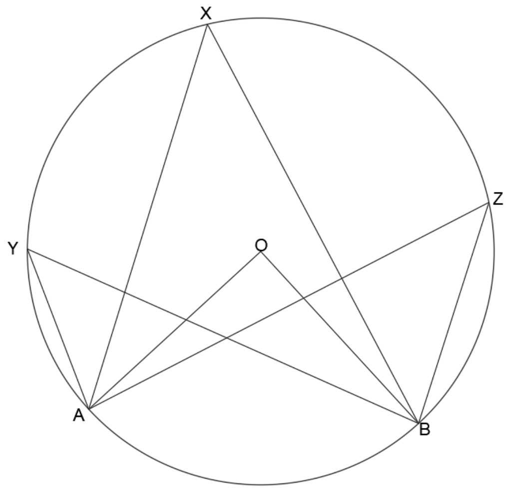
Theorem 5: There is one and only one circle passing through three non-collinear points

Theorem 6: Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres)
Theorem 7: Chords equidistant from the centre of a circle are equal in length.

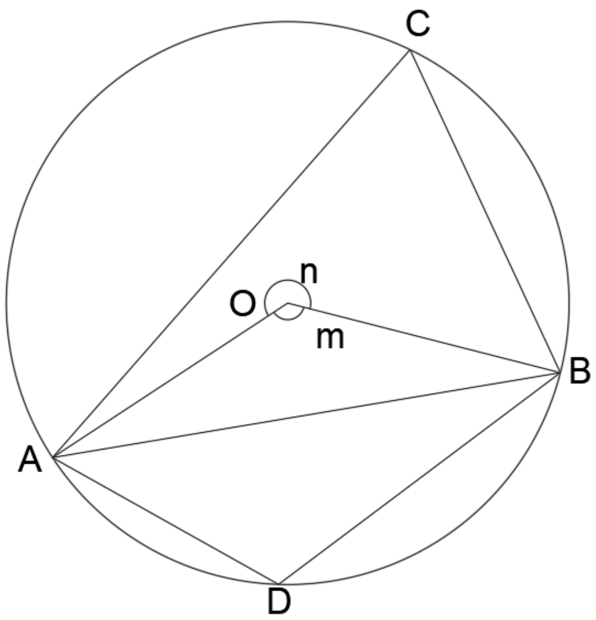


6. If PQ = RS, then OL = OM
7. If OL = OM, then PQ = RS

Theorem 8: The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle
Theorem 9: Angles in the same segment of a circle are equal



8. $\angle AOB = 2\angle AXB = 2\angle AYB = 2\angle AZB$
9. $\angle AXB = \angle AYB = \angle AZB$

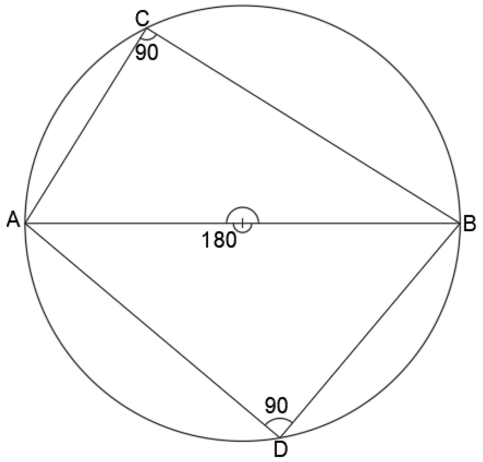


Angle subtended at the centre is $m = \angle AOB$
Its reflex angle is $n = 360 - m$

* Angle subtended at the circumference on a point (C) in the *same segment* (ACB) is half the angle subtended at the centre.
It means that $\angle ACB = \frac{1}{2}\angle AOB$ -or- $\angle AOB = 2\angle ACB$
Chord AB is the common base for triangles AOB and ACB in the same segment.

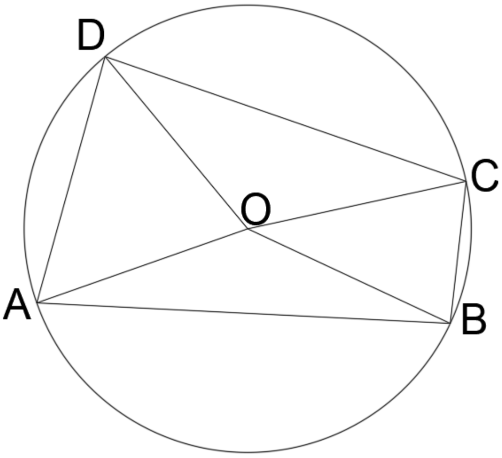
* Angle subtended at the circumference on a point (D) in the *other segment* (ADB) is half the *reflex angle* subtended at the centre.
It means that $\angle ADB = \frac{1}{2} reflex \angle AOB = \frac{1}{2} n = \frac{1}{2}(360 - m)$
Chord AB is the common base for triangles AOB and ADB is different segments.

Angle subtended by the diameter is 90°



AB is the diameter. Angle AOB = 180°. Angles on the circumference are half the angles subtended at the centre. Therefore, ACB = 90°. ADB = 90°.

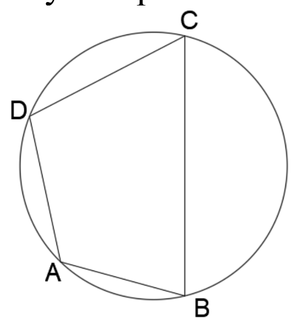
Triangles with one vertex at the centre are isosceles triangles.

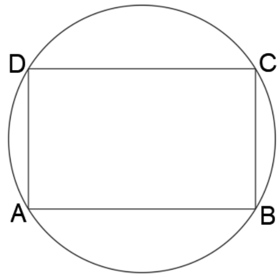


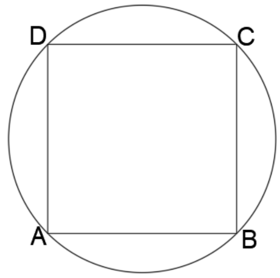
Triangles AOB, BOC, COD, DOA are isosceles triangles. Two sides of all these triangles are the radii.

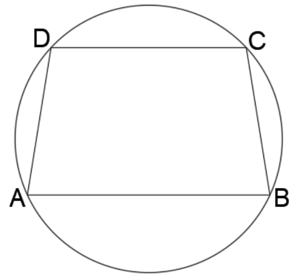
Cyclic quadrilateral

If all the vertices of a quadrilateral are on the circumference of a circle, it called a cyclic quadrilateral.



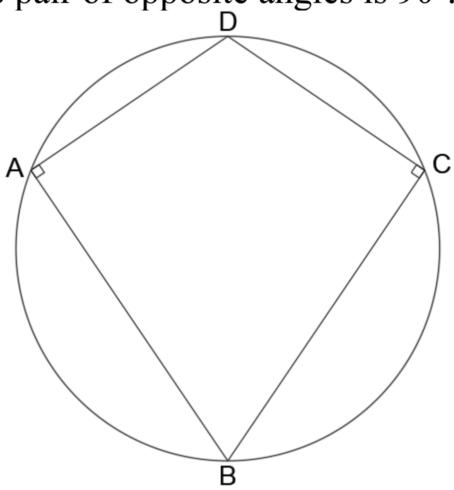






Rectangle, square, and isosceles trapezium are cyclic quadrilaterals.

Kites can be cyclic if one pair of opposite angles is 90^o.

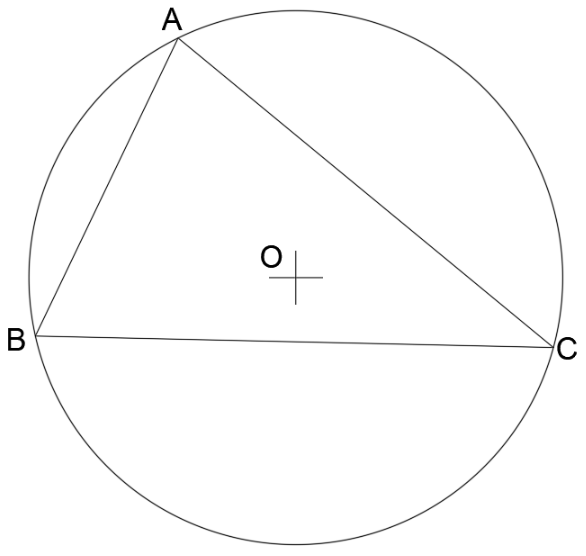


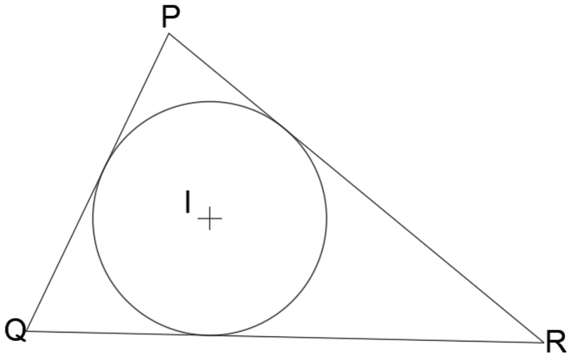
In any cyclic quadrilateral, the sum of the opposite angles adds to 180°.
Angles A + C = 180°
Angles B + D = 180°

Parallelogram and rhombus cannot be cyclic quadrilaterals. Opposite angles are equal in both. For them to be cyclic, opposite angles must add up to 180.
x + x = 180, or 2x = 180. x = 90. They can be cyclic only if parallelogram is a rectangle, or rhombus is a square.

The circle passing through the vertices of a triangle ABC is called Circumcircle. Its centre O is called circumcentre. It is equidistant from vertices A, B, C.

The circle touching all the sides of a triangle PQR is called Incircle. Its centre I is called the incentre. It is equidistant from sides PQ, PR, QR.



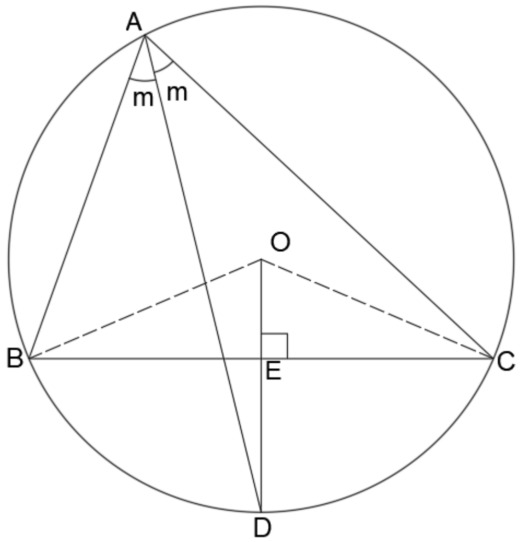


The properties that are used while solving circles exercises are:

- Angle subtended by a chord at centre is double the angle subtended by the same chord on the circumference in the same segment
- All angles subtended by a chord on the circumference in the same segment are equal
- Triangle formed when one side is the diameter is a right-angled triangle
- Opposite angles in a cyclic quadrilateral add to 180°
- Triangles with one vertex at the centre are isosceles. Two sides and two angles are equal.
- Triangles formed using a radius, half a chord, and line from centre to chord are right-angled triangles
- Perpendicular bisectors of all sides of a triangle meet at a single point called the circumcentre. Centre-to-vertex is the radius

- Angle bisectors of all angles of a triangle meet at a single point called the incentre.
- Congruency property (SSS, SAS, ASA, AAS, RHS)
- Parallel line property (alternate interior angles, co-interior, corresponding angles)
- Pythagoras theorem

Example:



In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that the intersection of these two lines is on the circumference, D.

a. Join BO and CO.

b. BC is common base for triangles BOC and BAC.
BAC = 2m
Therefore, BOC = 4m (Angle at centre **double** the angle on circumference)

c. The perpendicular bisector of a chord passes through the centre. O is the **circumcentre** of the circle. OB, OC are the radii

d. In triangles OEB, OEC –
OB = OC (Side)
$\llcorner OEB = \llcorner OEC = 90°$ (Angle)
OE is common (Side)

From SAS **congruency** rule, $\Delta OEB \cong \Delta OEC$
Therefore, $\angle BOE = \angle COE$

$\angle BOC = \angle BOE + \angle COE = 2\angle BOE$
$\angle BOC = 4m$
Therefore, $\angle BOE = \angle COE = 2m$

e. $\angle COE = \angle COD = 2m$
$\angle CAD = m$
A is on the circumference. Angle on circumference is **half** the angle made at the centre. This is possible only is CD is a common chord, that is, D is on the circumference.

f. Similarly for angles BAD and BOD, the angles are m and 2m respectively. This is possible only if BD is a common chord for both triangles. Therefore, D is on the circumference.