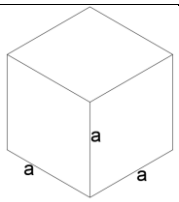
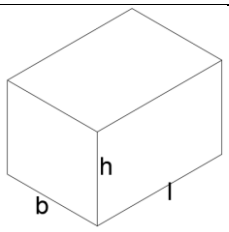
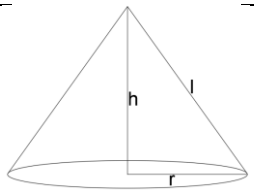
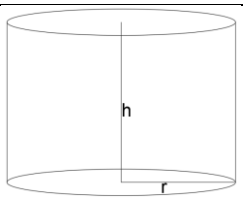
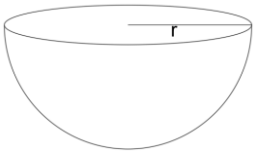
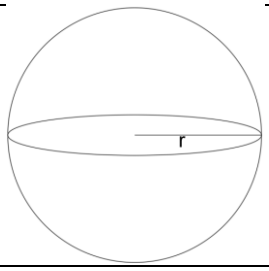
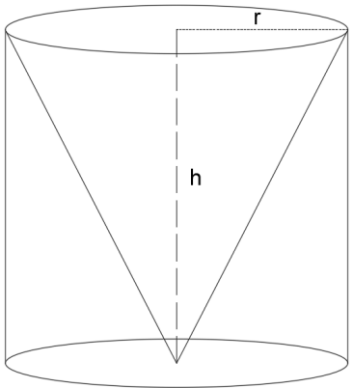
Surface Area and Volumes of Solids

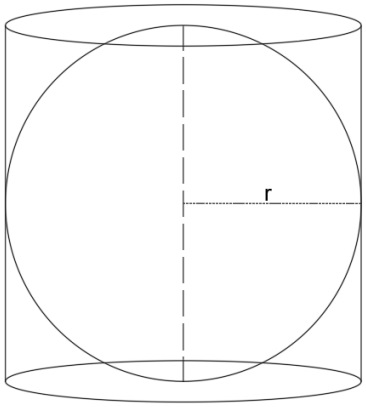
Solid		Lateral/Curved Surface Area	Total Surface Area	Volume
Cube		$4a^2$	$6a^2$	a^3
Cuboid		$2(bh + lh)$	$2(bh + hl + lb)$	lbh
Cone		πrl	$\pi r(r + l)$	$\frac{1}{3}\pi r^2 h$
		$l = \sqrt{h^2 + r^2}$		
Cylinder		$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$
Hemisphere		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
Sphere		$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$



For the same radius and height, volume of a cone is <u>one-third</u> the volume of a cylinder.

Volume of cylinder = $\pi r^2 h$

Volume of cone = $\frac{1}{3}\pi r^2 h$

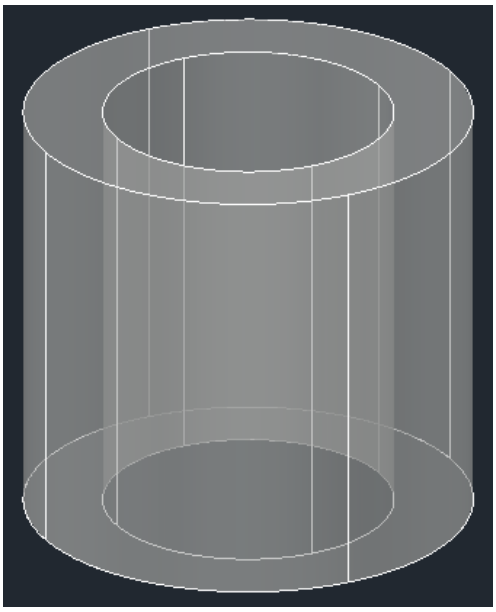


For the same diameter and height, surface area of a sphere and curved surface area of cylinder are the <u>same</u>.

Surface area of sphere = $4\pi r^2$

CSA of cylinder = $2\pi rh = 2\pi r(2r) = 4\pi r^2$

a. Hollow cylinder:

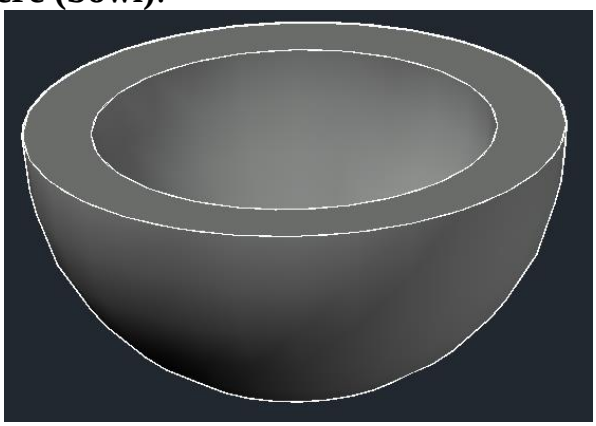


Outer circle radius: R (diameter D)
Inner circle radius: r (diameter d)
Height: h
Thickness = R – r = $\frac{D-d}{2}$

i. Volume of hollow cylinder = outer volume – inner volume
Volume = $\pi R^2 h - \pi r^2 h$
Volume of hollow cylinder = $\boldsymbol{\pi h(R^2 - r^2)}$

ii. Total surface area = CSA of outer + CSA of inner + area of bottom disc + area of top disc
CSA of outer cylinder = $2\pi Rh$
CSA of inner cylinder = $2\pi rh$
Total areas of top and bottom flat discs = $2 \times (\pi R^2 - \pi r^2) = 2\pi(R^2 - r^2)$
Total surface area of hollow cylinder = $\boldsymbol{2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)}$

b. Hollow hemisphere (bowl):



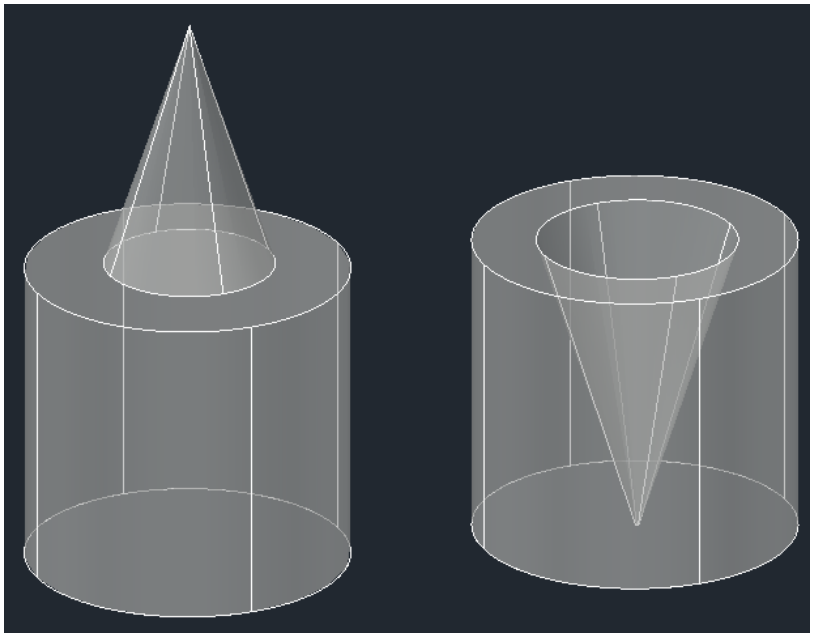
Outer radius: R
Inner radius: r

i. Volume of hollow hemisphere = outer volume – inner volume
Volume = $\frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$
Volume of hollow hemisphere= $\boldsymbol{\frac{2}{3}\pi(R^3 - r^3)}$

ii. Total surface area = CSA of outer + CSA of inner + area of disc
CSA of outer hemisphere = $2\pi R^2$
CSA of inner hemisphere = $2\pi r^2$
Area of flat disc = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$
Total surface area = $2\pi R^2 + 2\pi r^2 + \pi R^2 - \pi r^2$
Total surface area of hollow hemisphere = $\boldsymbol{3\pi R^2 + \pi r^2}$

Combined solids

a. **Cone and cylinder**



Cylinder radius: R
Cylinder height: H

Cone radius: r
Cone height: h
Cone slant height: $l = \sqrt{r^2 + h^2}$

i. Cone on cylinder:
Total volume of solid = volume of cylinder + volume of cone
Vol = $\pi R^2 H + \frac{1}{3}\pi r^2 h$

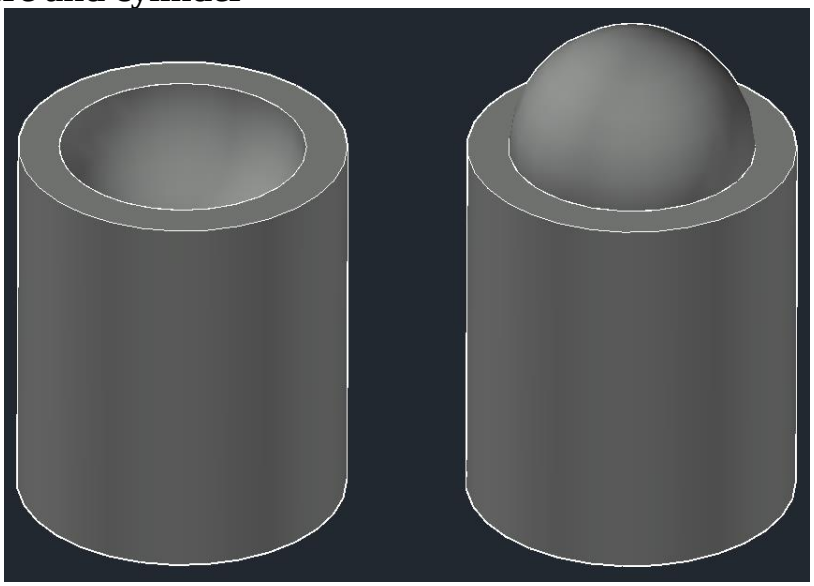
Total surface area = CSA of cylinder + base area of cylinder + disc area + CSA of cone
TSA = $2\pi RH + \pi R^2 + \pi(R^2 - r^2) + \pi rl$

ii. Cone inside cylinder:
Total volume of solid = volume of cylinder – volume of cone
Vol = $\pi R^2 H - \frac{1}{3}\pi r^2 h$

Total surface area = CSA of cylinder + base area of cylinder + disc area + CSA of cone
TSA = $2\pi RH + \pi R^2 + \pi(R^2 - r^2) + \pi rl$

Note: If radius of cone and cylinder are same, R = r, and disc area is zero.

b. **Hemisphere and cylinder**



Cylinder radius: R
Hemisphere radius: r

i. Hemisphere on cylinder:
Total volume of solid = volume of cylinder + volume of hemisphere
Vol = $\pi R^2 h + \frac{2}{3}\pi r^3$

Total surface area = CSA of cylinder + base area + disc area + CSA of hemisphere
TSA = $2\pi Rh + \pi R^2 + \pi(R^2 - r^2) + 2\pi r^2$
TSA = $2\pi Rh + 2\pi R^2 + \pi r^2$

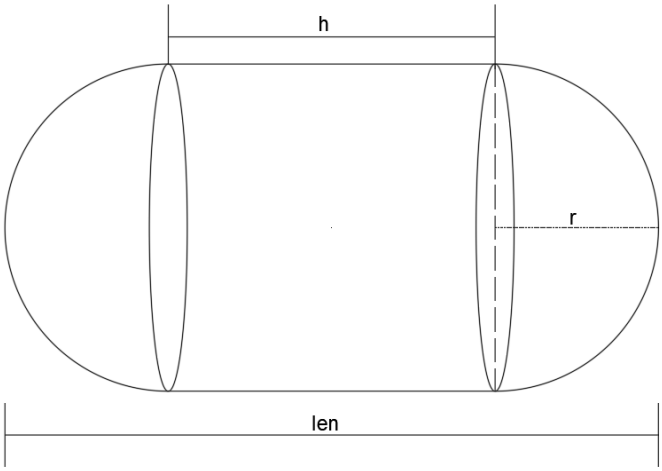
ii. Hemisphere inside cylinder:
Total volume of solid = volume of cylinder - volume of hemisphere
Vol = $\pi R^2 h - \frac{2}{3}\pi r^3$

Total surface area = CSA of cylinder + base area + disc area + CSA of hemisphere
TSA = $2\pi Rh + \pi R^2 + \pi(R^2 - r^2) + 2\pi r^2$
TSA = $2\pi Rh + 2\pi R^2 + \pi r^2$

Note: If radius of hemisphere and cylinder are same, R = r, and disc area is zero.

c. **Two hemispheres on cylinder**

A solid is made of two hemispheres on both flat surfaces of a cylinder. Total length of the solid is len.



Height of cylinder, h = len – 2r

Total volume = volume of two hemispheres + volume of the cylinder
Volume of two hemispheres = volume of whole sphere
Vol = $\frac{4}{3}\pi r^3 + \pi r^2 h$

Surface area of the solid = CSA of two hemispheres + CSA of cylinder
CSA of two hemisphere = surface area of whole sphere
Surface area of the solid = $4\pi r^2 + 2\pi r h$